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Test of the universal rise of hadronic total cross sections at super-high energies

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Abstract. The increase of the total cross sections at very high energies described by $\log^2(s/s_0)$ appears to be confirmed. In the analysis of the COMPETE collaboration in the Particle Data Group (2006), the $B\log^2(s/s_0)$ was assumed to extend the universal rise of all the total hadronic cross sections to reduce the number of adjustable parameters. We test if the assumption on the universality of B is justified, through investigation of the values of B for $\pi^{\pm}p(K^{\pm}p)$ and $\bar{p}p, pp$ scatterings. We search for the simultaneous best fit to the $\sigma_{\rm tot}$ and ρ ratios, using a constraint from the FESR of the P' type for $\pi^{\mp}p$ scatterings and constraints that are free from the unphysical regions for the $\bar{p}p, pp$ and $K^{\pm}p$ scatterings. By including rich information of the low-energy scattering data owing to the use of FESR, the errors of the B parameters decrease especially for πp. The resulting value of B_{pp} is consistent with $B_{\pi p}$ within two standard deviations, which appears to support the universality hypothesis.

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1 Purpose of this paper

It is well known as the Froissart unitarity bound [1, 2], that the increase of the total cross sections is at most $\log^2\nu$. It had not been possible, however, to discriminate between asymptotic $\log\nu$ and $\log^2\nu$ fits if one uses πN high-energy data alone above 70 GeV. Therefore, we have proposed [3] to use the rich information of the πp total cross sections at low- and intermediate-energy regions by means of the finite-energy sum rules (FESR) of the P' type [4], as well as [5, 6] in addition to the total cross sections, and we have arrived at the conclusion that $\log^2\nu$ behavior is preferred. Cudell et al. (COMPETE collaboration) [7] have considered several classes of analytic parametrizations of hadronic scattering amplitudes, and they have compared their predictions to all available forward data (pp, $\bar{p}p$, πp, Kp, γp, $\gamma\gamma$ and $\Sigma^- p$). Although these parametrizations were very close for $\sqrt{s}\geq 9$ GeV, it turned out that they differ markedly at low energy, where $\log^2 s$ enables one to extend the fit down to $\sqrt{s}=4$ GeV [7].

The statement that the $\log^2\nu$ behavior is preferred has been confirmed in [8–10]. In [7], the $B\log^2(s/s_0)$ was assumed to extend the universal rise of all the total hadronic cross sections. This resulted in reducing the number of adjustable parameters. Recently, however, it was pointed out in [11] that [8–10] gave different predictions for the value of B for πN and NN, i.e., different predictions at super-high energies: $\sigma^{as}_{\pi N} > \sigma^{as}_{NN}$ [8] and $\sigma^{as}_{\pi N}\sim 2/3\sigma^{as}_{NN}$ [9, 10].

The purpose of this article is to investigate the value of B for the $\pi^{\pm}p(K^{\pm}p)$ and $\bar{p}p, pp$ cases in order to check if the assumption of the universality of the coefficient B is justified. We search for the simultaneous best fit to $\sigma_{\rm tot}$, the total cross sections, and ρ, the ratio of the real to imaginary part of the forward scattering amplitude, using a constraint from the FESR of the P' type for $\pi^{\pm}p$ scatterings and constraints that are free from the unphysical regions for $\bar{p}p, pp$ and $K^{\pm}p$ scatterings [12].

2 Total cross sections, ρ ratios and constraints

Let us consider the forward $\bar{p}p, pp$, $\pi^{\mp}p$ and $K^{\mp}p$ scatterings. We take both the crossing-even and crossing-odd forward scattering amplitudes, $F^{(+)}$ and $F^{(-)}$, defined by

$$F^{(\pm)}(\nu)=\frac{f^{\bar{a}p}(\nu)\pm f^{ap}(\nu)}{2}\,, \qquad (1)$$

$$f^{\bar{a}p}(\nu)=F^{(+)}(\nu)+F^{(-)}(\nu)\,,$$
$$f^{ap}(\nu)=F^{(+)}(\nu)-F^{(-)}(\nu)\,, \qquad (2)$$

where $(\bar{a},a)=(\bar{p},p)$, (π^-,π^+) and (K^-,K^+), respectively, and ν is the incident energy of $p(\bar{p})$, π and K in the labora-

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tory system. We assume that

$$\begin{aligned}\operatorname{Im} F^{(+)}(\nu) &\simeq \operatorname{Im} R(\nu)+\operatorname{Im} F_{P'}(\nu)\\ &=\frac{\nu}{m^2}\left(c_0+c_1\log\frac{\nu}{m}+c_2\log^2\frac{\nu}{m}\right)\\ &\quad+\frac{\beta_{P'}}{m}\left(\frac{\nu}{m}\right)^{\alpha_{P'}}, \quad (3)\end{aligned}$$

$$\operatorname{Im} F^{(-)}(\nu) \simeq \operatorname{Im} F_V(\nu)=\frac{\beta_V}{m}\left(\frac{\nu}{m}\right)^{\alpha_V} \quad (4)$$

at high energies for $\nu > N$. Here $m = M$ (proton mass), $m = \mu$ (pion mass) and $m = m_K$ (kaon mass) for $\bar{p}(p)p$, πp and Kp scatterings, respectively. Using the crossing-even/odd property, $F^{(\pm)}(-\nu) = \pm F^{(\pm)}(\nu)^*$, the real parts are given by [3, 9, 10]

$$\begin{aligned}\operatorname{Re} F^{(+)}(\nu) &\simeq \frac{\pi\nu}{2m^2}\left(c_1+2c_2\ln\frac{\nu}{m}\right)\\ &\quad-\frac{\beta_{P'}}{m}\left(\frac{\nu}{m}\right)^{\alpha_{P'}}\cot\frac{\pi\alpha_{P'}}{2}+F^{(+)}(0), \quad (5)\end{aligned}$$

$$\operatorname{Re} F^{(-)}(\nu) \simeq \frac{\beta_V}{m}\left(\frac{\nu}{m}\right)^{\alpha_V}\tan\frac{\pi\alpha_V}{2}. \quad (6)$$

The total cross sections $\sigma_{\text{tot}}^{\bar{a}p}$, σ_{tot}^{ap} and the ρ ratios $\rho^{\bar{a}p}$ and ρ^{ap} are given by

$$\begin{aligned}\operatorname{Im} f^{\bar{a}p,ap}(\nu) &= \frac{k}{4\pi}\sigma_{\text{tot}}^{\bar{a}p,ap},\\ \rho^{\bar{a}p} &= \frac{\operatorname{Re} f^{\bar{a}p}}{\operatorname{Im} f^{\bar{a}p}}, \quad \rho^{ap}=\frac{\operatorname{Re} f^{ap}}{\operatorname{Im} f^{ap}}, \quad (7)\end{aligned}$$

respectively, where the k are the incident momenta of $p(\bar{p})$, π and K in the laboratory system.

Defining $\tilde{F}^{(+)}(\nu) = F^{(+)}(\nu) - R(\nu) - F_{P'}(\nu) - F^{(+)}(0) \sim \nu^{\alpha(0)}$ $(\alpha(0) < 0)$, for large value of ν, we have obtained the FESR [3] in the spirit of the P' sum rule [4]

$$\begin{aligned}\operatorname{Re}\tilde{F}^{(+)}(m) &= \frac{2P}{\pi}\int_0^m \frac{\nu}{k^2}\operatorname{Im} F^{(+)}(\nu)\,\mathrm{d}\nu\\ &\quad+\frac{1}{2\pi^2}\int_0^{\overline{N}}\sigma_{\text{tot}}^{(+)}(k)\,\mathrm{d}k\\ &\quad-\frac{2P}{\pi}\int_0^N\frac{\nu}{k^2}\left\{\operatorname{Im} R(\nu)+\operatorname{Im} F_{P'}(\nu)\right\}\mathrm{d}\nu, \quad (8)\end{aligned}$$

where $\overline{N} = \sqrt{N^2 - m^2} \simeq N$. The $\overline{N}$ is taken to be $\overline{N} = 20\,\text{GeV}$ for πp scattering, which is sufficiently large in the asymptotic region. Let us call (8) FESR(1) (0–$\overline{N}$).[1]

[1] This sum rule should hold if no singularities extend above $J = 0$ except for the pomeron and P'. So, we adopt this sum rule rather than the higher-moment sum rule. As was shown by one of the authors (K.I.) in [4], the dominance of pomeron and P' gives phenomenologically good results. So we took the standpoint of Regge pole dominance; i.e., we assumed there to be no Regge cuts as the simplest assumption.

Equation (8) gives directly a constraint for πp scattering:

$$\begin{aligned}&\frac{2P}{\pi}\int_0^N\frac{\nu}{k^2}\left\{\operatorname{Im} R(\nu)+\operatorname{Im} F_{P'}(\nu)\right\}\mathrm{d}\nu\\ &\quad-\operatorname{Re} R(\mu)-\operatorname{Re} F_{P'}(\mu)\\ &=-\operatorname{Re} F^{(+)}(\mu)+(\text{pole term})+\frac{1}{2\pi^2}\int_0^{\overline{N}}\sigma_{\text{tot}}^{(+)}(k)\,\mathrm{d}k. \quad (9)\end{aligned}$$

For πp scattering, the RHS can be estimated with sufficient accuracy, regarding (9) as an exact constraint [3]:

$\operatorname{Re} F^{(+)}(\mu)$ is represented by the scattering lengths, and the pole term comes only from the nucleon. The last term is estimated from the rich data of the experimental $\sigma_{\text{tot}}^{\pi^{\mp}p}$.

On the other hand, (8) for $\bar{p}(p)p$ scattering suffers from the unphysical regions coming from boson poles below the $\bar{p}p$ threshold. Reliable estimates, however, are difficult to obtain. Similarly in Kp scattering, poles of the Λ and Σ resonant states contribute below the K^-p threshold. In [12], we have presented a new constraint, free from unphysical regions. We consider (9) with $N = N_1$ and $N = N_2$ $(N_2 > N_1)$. Taking the difference between these two relations, we obtain the relation

$$\begin{aligned}&\frac{2}{\pi}\int_{N_1}^{N_2}\frac{\nu}{k^2}\left\{\operatorname{Im} R(\nu)+\operatorname{Im} F_{P'}(\nu)\right\}\mathrm{d}\nu\\ &=\frac{1}{2\pi^2}\int_{\overline{N}_1}^{\overline{N}_2}\sigma_{\text{tot}}^{(+)}(k)\,\mathrm{d}k. \quad (10)\end{aligned}$$

The RHS can be estimated from the experimental $\sigma_{\text{tot}}^{\bar{p}p,pp}$ and $\sigma_{\text{tot}}^{K^{\mp}p}$ data,[2] regarding (10) as an exact constraint. Equation (10) is called FESR(1) ($\overline{N_1}$–$\overline{N_2}$). In the actual analyses of $p(\bar{p})p$ and Kp, $\overline{N_1}(\overline{N_2})$ is taken to be 10(20) GeV.

3 The general approach

The formulae (1)–(7) and the constraints (9) and (10) are our starting points. The $\sigma_{\text{tot}}^{\bar{a}p,ap}$ and $\rho^{\bar{a}p,ap}$ are fitted simultaneously for the respective processes of the $\bar{p}(p)p$, πp and Kp scatterings. The high-energy parameters c_2, c_1, c_0, $\beta_{P'}$ and β_V are treated as process-dependent, while $\alpha_{P'}$ and α_V are fixed with common values for every process. The FESR(1) (10–20 GeV) ((10) with $\overline{N}_1(\overline{N}_2) = 10(20)$ GeV) and FESR(1) (0–20 GeV) ((9) with $\overline{N} = 20$ GeV) give constraints between c_2, c_1, c_0 and $\beta_{P'}$ for $\bar{p}(p)p$, Kp and πp scatterings, respectively. $F^{(+)}(0)$ is treated as an additional parameter, and the number of fitting parameters is 5 for each process. The resulting c_2 are related to the B parameters, defined by $\sigma \simeq B\log^2(s/s_0) + \cdots$, through the

[2] Practically it is estimated from the fit to σ_{tot} in 2.5 GeV $\le k \le$ 100 GeV through the phenomenological formula [12].

Table 1. Values of parameters and χ^2 in the best fits in $(\alpha_{P'}, \alpha_V) = (0.500, 0.497)$. Both total χ^2 and respective χ^2 for all data with the number of data points are given. The result of $\bar{p}(p)p$ scattering is in the first row, πp scattering in the second row and Kp scattering in the third row. The errors are given only for c_2. The values of $\beta_{P'}$ are obtained from FESR. $F^{(+)}(0)$ is given in GeV^{-1}

	c_2	c_1	c_0	$F^{(+)}(0)$	β_V	$\beta_{P'}$	$\frac{\chi^2_{\rm tot}}{N_D - N_P}$	$\frac{\chi^{2,\sigma}_{\bar{a}p}}{N^\sigma_{\bar{a}p}}$	$\frac{\chi^{2,\rho}_{\bar{a}p}}{N^\rho_{\bar{a}p}}$	$\frac{\chi^{2,\sigma}_{ap}}{N^\sigma_{ap}}$	$\frac{\chi^{2,\rho}_{ap}}{N^\rho_{ap}}$
$\bar{p}p, pp$	0.0520 ± 0.0041	-0.259	6.67	11.1	3.75	6.82	$\frac{157.3}{231-5}$	$\frac{21.0}{51}$	$\frac{15.4}{16}$	$\frac{61.4}{94}$	$\frac{59.5}{70}$
$\pi^\mp p$	$(140 \pm 14) \times 10^{-5}$	-0.0153	0.132	0.291	0.0392	0.0875	$\frac{42.3}{151-5}$	$\frac{10.3}{84}$	$\frac{11.7}{22}$	$\frac{8.3}{37}$	$\frac{12.0}{8}$
$K^\mp p$	0.0185 ± 0.0103	-0.142	1.20	1.86	0.573	0.234	$\frac{36.3}{111-5}$	$\frac{17.9}{53}$	$\frac{11.5}{13}$	$\frac{5.9}{31}$	$\frac{1.0}{14}$

equation

$$B_{ap} = \frac{4\pi}{m^2} c_2\,, \quad m = M, \mu, m_K\,, \quad a = p, \pi, K\,, \quad (11)$$

and we can test the universality of the B parameters for the relevant processes.

4 Result of the analyses

The $\sigma^{\bar{a}p,ap}_{\rm tot}$ for $k \geq 20\,\mathrm{GeV}$ and $\rho^{\bar{a}p,ap}$ for $k \geq 5\,\mathrm{GeV}$ are fitted simultaneously.[3] Here we choose $k \geq 20\,\mathrm{GeV}$ as a fitted energy region of the σ data, being different from the ρ data, since the σ data up to $k = 20\,\mathrm{GeV}$ are already used to obtain their integrals appearing in FESR, (9) and (10). We take two cases, $(\alpha_{P'}, \alpha_V) = (0.500, 0.497)$ and $(0.542, 0.496)$. These values are selected by considering the χ^2 behaviors of the fit to the $\bar{p}p, pp$ data: the total χ^2 is almost independent of the input value of $\alpha_{P'}$, while it is sensitive to the value of α_V. So we select two values of $\alpha_{P'}$ as typical examples,[4] while α_V is selected from the minima of χ^2. The χ^2 takes its minimum at $\alpha_V \sim 0.50$ independently of the $\alpha_{P'}$-value.

The FESR(1) (10–20 GeV), (10) for $\bar{p}(p)p, Kp$, and FESR(1) (0–20 GeV), (9) for πp, are given respectively by

$$1.837(2.061)\beta_{P'} + 7.247 c_0 + 19.96 c_1 + 55.27 c_2 = 58.54\,, \quad (12)$$

$$4.810(5.542)\beta_{P'} + 26.14 c_0 + 88.74 c_1 + 302.3 c_2 = 25.41\,, \quad (13)$$

$$109.2(124.1)\beta_{P'} + 653.6 c_0 + 2591 c_1 + 10928 c_2 = 71.12\,, \quad (14)$$

where the number without (with) parentheses of the $\beta_{P'}$ coefficient is the case of $\alpha_{P'} = 0.500(0.542)$. Solving the above equations for $\beta_{P'}$, they are represented by the other three parameters as $\beta_{P'} = \beta_{P'}(c_2, c_1, c_0)$, and the fitting parameters are c_2, c_1, c_0, β_V and $F^{(+)}(0)$ for the respective processes.

The results of the fits are depicted in Fig. 1a and b for $\bar{p}(p)p$ scattering, c and d for πp scattering and e and f for Kp scattering, respectively. The values of the parameters and χ^2 in the best fits are summarized in Table 1.

There are several comments to be made in the analyses. The fit to the original $\bar{p}p, pp$ data in Particle Data Group 2006 [11] gives the total $\chi^2/(N_D - N_P) = 224.8/(240-5)$, and the χ^2/N_D for the respective data of $(\sigma^{\bar{p}p}, \rho^{\bar{p}p}, \sigma^{pp}, \rho^{pp})$ are $(20.9/51, 15.2/16, 62.4/94, 126.3/79)$. Here the fit to the ρ^{pp} data is unsuccessful, reflecting the situation that the ρ^{pp} data are mutually inconsistent with different experiments. In Fig. 1b Fajardo 80 [13] (red points) and Bellettini 65 [14] (orange points) have comparatively small errors, and these points seem to be inconsistent with the other points by inspection. We have tried to fit the data set only including Fajardo 80 for ρ^{pp} in the relevant energy region, but it is not successful. We remove these two data from our fit given in Table 1.

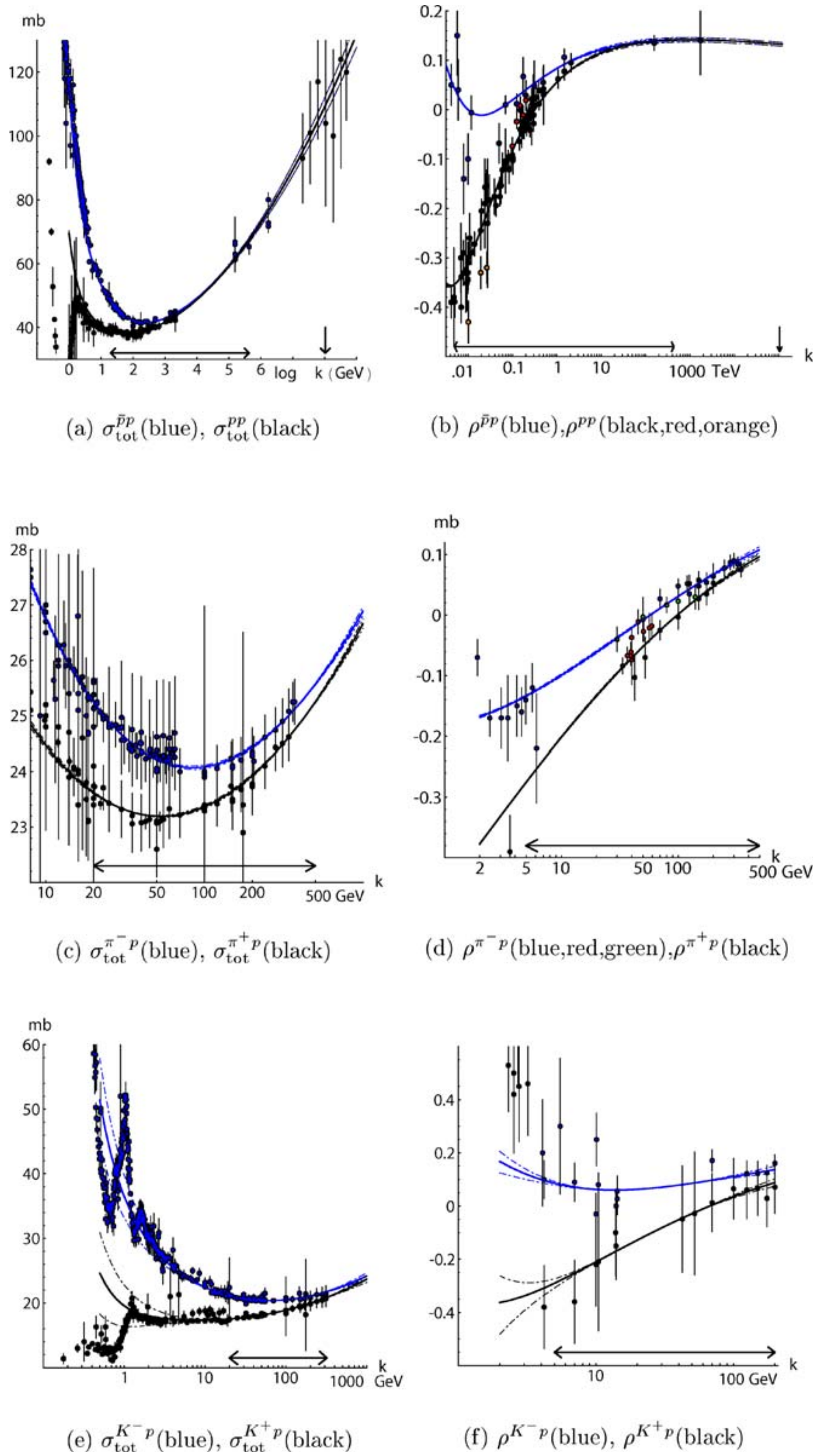
A similar situation occurs for πp. The fit to the original data gives the total $\chi^2/(N_D - N_P) = 73.8/(162-5)$, and the χ^2/N_D for the respective data of $(\sigma^{\pi^- p}, \rho^{\pi^- p}, \sigma^{\pi^+ p}, \rho^{\pi^+ p})$ are $(11.5/84, 42.4/33, 7.3/37, 12.7/8)$. The fit to the $\rho^{\pi^- p}$ data is unsuccessful. In Fig. 1d Apokin [15–17] (red points) in $30\,\mathrm{GeV} \leq k \leq 60\,\mathrm{GeV}$, which have small errors, are inclined to give smaller values (which are almost in the region of the $\rho^{\pi^+ p}$ data!) than the other data, by Burq [18] (green points). We try to fit the data excluding Apokin [15–17] (named the Burq fit) and to fit the data excluding Burq [18] (named the Apokin fit). The total χ^2 gives respectively $42.25/(151-5)$ and $69.8/(156-5)$ for the Burq and Apokin fit, while the χ^2/N_D for $\rho^{\pi^- p}$ are 11.7/22 and 37.8/27. Thus, only the Burq fit is successful. The Apokin fit is almost the same as the fit to the original data, and the $\rho^{\pi^- p}$ data around $k = 5\,\mathrm{GeV}$ are not reproduced well in both fits,

Table 2. Values of the B parameters in mb. The results are given in the two cases $\alpha_{P'} = 0.500$ and 0.542

process	B	$\alpha_{P'} = 0.500$	$\alpha_{P'} = 0.542$
$\bar{p}p, pp$	B_{pp}	0.289 ± 0.023	0.268 ± 0.024
$\pi^\mp p$	$B_{\pi p}$	0.351 ± 0.036	0.333 ± 0.039
$K^\mp p$	B_{Kp}	0.37 ± 0.21	0.37 ± 0.22

[3] In the actual analysis we fit data of $\mathrm{Re}\, f$ instead of ρ. $\mathrm{Re}\, f$ data are obtained from the original ρ data multiplied by $\sigma_{\rm tot}$, which is given by the fit in [11].

[4] Our $\alpha_{P'}$ corresponds to $1 - \eta_1$ in the parametrization of the COMPETE collaboration [11]. $\alpha_{P'} = 0.542$ corresponds to their best fit value, $\eta_1 = 0.458$.



(a) $\sigma_{\rm tot}^{\bar{p}p}$(blue), $\sigma_{\rm tot}^{pp}$(black)

(b) $\rho^{\bar{p}p}$(blue),ρ^{pp}(black,red,orange)

(c) $\sigma_{\rm tot}^{\pi^- p}$(blue), $\sigma_{\rm tot}^{\pi^+ p}$(black)

(d) $\rho^{\pi^- p}$(blue,red,green),$\rho^{\pi^+ p}$(black)

(e) $\sigma_{\rm tot}^{K^- p}$(blue), $\sigma_{\rm tot}^{K^+ p}$(black)

(f) $\rho^{K^- p}$(blue), $\rho^{K^+ p}$(black)

Fig. 1. Results of the fits to **a** $\sigma^{\bar{p}p,pp}$, **b** $\rho^{\bar{p}p,pp}$, **c** $\sigma^{\pi^\mp p}$, **d** $\rho^{\pi^\mp p}$, **e** $\sigma^{K^\mp p}$ and **f** $\rho^{K^\mp p}$. In **b**, *red (orange) points* Fajardo 80 [13] (Bellettini [14]) of ρ^{pp}. In **d**, *red (green) points* are Apokin [15–17] (Burq [18]) and *blue points* are the other ones in $\rho^{\pi^- p}$ data. The input-energy regions are shown by *horizontal arrows*. The LHC energy ($\sqrt{s} = 14$ TeV) is shown by *vertical arrows* in **a** and **b**

while they are well described in the Burq fit. By considering these facts we adopt the result of the Burq fit in Table 1.

By using (11), we can derive the B parameters from c_2 in Table 1. The result is given in Table 2 in the two cases $\alpha_{P'} = 0.500$ and 0.542. As seen in Table 2, B_{pp} is somewhat smaller than the $B_{\pi p}$, but it is consistent within two standard deviations, although its central value changes slightly, depending upon the choice of $\alpha_{P'}$. The central value of B_{Kp} is consistent with $B_{\pi p}$, although its error is very large, due to the present situation of the Kp data. Based on these results, the present experimental data are consistent with the hypothesis of the universal rise of the total cross section in super-high energies. On the other hand, $\sigma_{\pi N}^{as} \sim 2/3\, \sigma_{NN}^{as}$ [9, 10] appears not to be favored in our analysis. This is our main result.

5 Remarks on the analysis of πp

In order to obtain the above conclusion, it is essential to determine c_2 in πp (or $B_{\pi p}$) with enough accuracy. However, it is a very difficult task, since the experimental $\sigma_{\text{tot}}^{\pi p}$ are reported only in the very limited regions with momenta $k < 400$ GeV, in contrast with the $\sigma_{\text{tot}}^{\bar{p}p}$ data obtained up to $k = 1.7266 \times 10^6$ GeV. Actually, if we fit the same data in the fit of Table 1, using 6 (not 5) parameters with no use of the FESR, (14), we obtain

$$c_2 = (120 \pm 46) \times 10^{-5} \to B_{\pi p} = 0.301 \pm 0.116\,\text{mb}\,, \tag{15}$$

where $(\alpha_{P'}, \alpha_V) = (0.500, 0.497)$. The above value is consistent with the one given in Table 2, $B_{\pi p} = 0.351 \pm 0.036$ mb, within its large error. However, this error is very large, and the $B_{\pi p}$ in (15) is consistent with both $B_{pp}(= 0.289\,\text{mb})$ and $2/3\ B_{pp}(= 0.193\,\text{mb})$. So by using this value we cannot obtain any definite conclusion. In other words, *by including the rich information of the low-energy πp scattering data through FESR, the error of $B_{\pi p}$ is reduced to be less than one third ($0.116\,\text{mb} \to 0.036\,\text{mb}$), and as a result, the universality of B ($B_{pp} = B_{\pi p}$) appears to be preferred.*

In our analysis of Table 1, $\sigma_{\text{tot}}^{\pi^\mp p}$ in $k \geq \overline{N}_2$ and $\rho^{\pi^\mp p}$ in $k \geq 4.95$ GeV were fitted simultaneously, using FESR(1) (0–$\overline{N}_2$) with $\overline{N}_2 = 20$ GeV. When we analyze the data by taking $\overline{N}_2 = 25(30)$ GeV, the $B_{\pi p}$ are determined as $0.315 \pm 0.052 (0.303 \pm 0.060)$ mb. The results are not so sensitive to the choice of $\overline{N}_2$, although their errors become slightly larger.

If we use the FESR(1) (10–20 GeV) (not (0–20 GeV)) also for πp, similarly to $\bar{p}p(pp)$ and Kp and fit the same data, we obtain $B_{\pi p} = 0.314 \pm 0.075$ mb, which is consistent with our result given in Table 2; but its error becomes about twice larger than the value in Table 2. In order to obtain a sufficiently small error of $B_{\pi p}$ it appears to be important to include the information of the low-energy scattering data with $0 \leq k \leq 10$ GeV through FESR.

Finally, we would like to add several remarks.

(i) Our B_{pp}, $B_{pp} = 0.289 \pm 0.023$ mb (in case $\alpha_{P'} = 0.500$), is consistent with the value of B by the COMPETE collaboration [11], 0.308 ± 0.010 mb, which is obtained by assuming the universality of B for various processes.

(ii) Our B_{pp} is also consistent with the value by Block and Halzen [9, 10], 0.2817 ± 0.0064 mb or 0.2792 ± 0.0059 mb (from the c_2 parameter in Table III of [9, 10]). The present value of our B_{pp} is located between the above two results.

(iii) Our predictions of σ_{tot}^{pp} and ρ^{pp} at LHC energy ($\sqrt{s} = 14$ TeV) are

$$\sigma_{\text{tot}}^{pp} = 109.5 \pm 2.8\,\text{mb}\,, \quad \rho^{pp} = 0.133 \pm 0.004\,. \tag{16}$$

This value is consistent with our previous ones, $\sigma_{\text{tot}} = 107.1 \pm 2.6$ mb and $\rho = 0.127 \pm 0.004$ [12], which were obtained through the analysis based on only the crossing-even amplitude, using restricted data sets. The values of (16) are also located between the predictions of the relevant two groups, $\sigma_{\text{tot}}^{pp} = 111.5 \pm 1.2_{\text{syst}}{}^{+4.1}_{-2.1}{}_{\text{stat}}$ mb, $\rho^{pp} = 0.1361 \pm 0.0015_{\text{syst}}{}^{+0.0058}_{-0.0025}{}_{\text{stat}}$ [19] and $\sigma_{\text{tot}}^{pp} = 107.3 \pm 1.2$ mb, $\rho^{pp} = 0.132 \pm 0.001$ [9, 10].

(iv) The fit to the πp data given in Table 1 gives the prediction at $k = 610$ GeV, $\sigma_{\text{tot}}^{\pi^- p} = 25.91 \pm 0.03$ mb (in case of $\alpha_{P'} = 0.500$),[5] which is consistent with the recent observation by the SELEX collaboration, $\sigma_{\text{tot}}^{\pi^- N} = 26.6 \pm 0.9$ mb [20].

(v) In order to investigate the universality of B, we confined ourselves only to hadron–nucleon scatterings. We considered virtual photon scattering, observed extensively by HERA [21–23], to be beyond the scope of this paper.

(vi) Finally we would like to emphasize the importance of precise measurements of the ratios ρ in $\bar{p}p$, pp, $\pi^\mp p$ and $K^\mp p$ scatterings at intermediate energies above $k \geq 5$ GeV for further investigations of B parameters.

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[5] At this energy, $\sigma_{\text{tot}}^{\pi^+ p}$ is predicted with 25.62 ± 0.03 mb. The difference $\sigma_{\text{tot}}^{\pi^- p} - \sigma_{\text{tot}}^{\pi^+ p} \simeq 0.3$ mb.

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